

Soluções dos exercícios de Análise Matemática IV

1. Equação diferencial de 1ª ordem

$$1.4 \quad x(t) = \frac{C}{t} + t, \quad C \in \mathfrak{R}$$

$$1.5 \quad x(t) = Ct^n + t^n e^t, \quad C \in \mathfrak{R}$$

$$1.6 \quad x(t) = e^{1/t^2}$$

$$1.7 \text{ a) } x^3(t) = \frac{C(1+t^2)^{3/2}}{t^3} - 1, \quad C \in \mathfrak{R}$$

$$\text{b) } x^2(t)e^{t^2/x^2} = K, \quad K \in \mathfrak{R}$$

$$\text{c) } \frac{t^2}{x^3} - \frac{1}{x} = C, \quad C \in \mathfrak{R}$$

$$1.8 \quad x(t) = \begin{cases} 2(1-e^{-t}) & \text{se } t \in [0,1] \\ 2e^{-t}(e-1) & \text{se } t > 1 \end{cases}$$

$$1.10 \quad x(t) = \frac{aC}{bC + e^{-at}} + t, \quad C \in \mathfrak{R} \quad \vee \quad x(t) = \frac{a}{b}$$

$$1.14 \quad f(t) = -2\cos t + C, \quad C \in \mathfrak{R}, \quad x^2(t) \left(\frac{1}{2}C - \cos t \right) = K, \quad K \in \mathfrak{R}$$

$$1.15 \quad a = 1, \quad e^{-t} \operatorname{sen} x + t = C, \quad C \in \mathfrak{R}$$

$$1.16 \quad x(t) = C \cos t - \cos^2 t, \quad C \in \mathfrak{R}$$

$$1.17 \text{ a) } h(t, x) = \frac{3}{4}xt - \frac{1}{8}t^2 + \varphi(x), \quad \varphi(x) \text{ função arbitrária de } x$$

$$\text{b) } x^{3/2}t - \frac{t^2}{2}x^{1/2} + \int 2x^{-1/2}\varphi(x)dx = C, \quad C \in \mathfrak{R} \text{ e } \varphi(x) \text{ função arbitrária de } x$$

$$1.18 \quad t^3x + tx^3 + 2t + 5x = 9$$

$$1.19 \text{ c) } 2x^2 + (2t-1)x + \frac{t^2}{2} + 3t = C, \quad C \in \mathfrak{R}$$

$$1.20 \text{ a) } \mu(t) = e^{\int a(t)dt}$$

$$\text{b) } x(t) = Ce^{-t^2} - 1, \quad C \in \mathfrak{R} \quad \lim_{t \rightarrow +\infty} x(t) = -1$$

$$1.21 \text{ a) } x^2(Ce^{t^2} + t^2 + 1) = 1, \quad C \in \mathfrak{R}$$

$$\text{b) } x(t) = \frac{2}{1 + 2Ce^{t^2}}, \quad C \in \mathfrak{R}$$

$$\text{c) } x(t) = \frac{tgt + \sec t}{sent + C}, \quad C \in \mathfrak{R}$$

$$\text{d) } x(t) = \frac{1}{C\sqrt{|t^2 - 1|} - s}, \quad C \in \mathfrak{R}$$

$$1.23 \quad x(t) = -e^{-t}$$

$$1.24 \quad x(t) = C + \frac{t^2}{2}, \quad C \in \mathfrak{R} \quad \vee \quad x(t) = \left(\frac{t}{A} - \frac{1}{A^2}\right)e^{At} + B, \quad A \neq 0 \wedge B \in \mathfrak{R}$$

$$1.26 \quad x(t) = 3e^{t^2} - 2$$

$$1.27 \text{ a) } x_1(t) = 1 + t + \frac{t^3}{3}, \quad x_2(t) = 1 + t + t^2 + \frac{2t^3}{3} + \frac{5t^4}{12} + \frac{2t^5}{15} + \frac{t^6}{18} + \frac{t^7}{63}$$

b) 10

$$\text{c) } x(t) = \frac{e^{t^3/3}}{1 - \int_0^t e^{l^3/3} dl}$$

$$1.28 \quad x(t) = 0 \quad \forall_{t \in \mathfrak{R}}, \quad \sqrt{x} = \frac{1}{2} \int_0^t g(s) ds \quad \text{s\~{a}o ambas solu\~{c}oes do PVI. Falha a condi\~{c}ao}$$

de Lipschitz.

$$1.29 \quad x(t) \equiv 1, \quad \int_1^x \frac{1}{\sqrt{s^2 - 1}} ds = t, \quad f \notin C^1([1, +\infty))$$

$$1.30 \quad \theta \in \left(0, \frac{1}{T}\right)$$

$$1.32 \text{ a) } x(t) = \frac{1}{2}(e^t + e^{-t}) = \cosh t$$

$$\text{b) PVI} \begin{cases} x'' = x \\ x(0) = 1 \\ x'(0) = 0 \end{cases}$$

2. Equação diferencial de ordem n

2.4

$$\text{a) } x(t) = -2e^{2t} + 3e^t$$

$$\text{b) } x(t) = (1-A)e^{-t} + (1-3t)Ae^{2t}, \quad A \in \mathfrak{R}$$

$$\text{c) } x(t) = -te^{-t} + e^{-t} \log|t+1| + te^{-t} \log|t+1| + C_1e^{-t} + C_2te^{-t}, \quad C_1, C_2 \in \mathfrak{R}$$

$$\text{d) } x(t) = C_1e^t + C_2e^{-2t} - \frac{2}{5} \cos 2t - \frac{6}{5} \text{sen} 2t, \quad C_1, C_2 \in \mathfrak{R}$$

$$\text{e) } x(t) = C_1 \cos 2t + C_2 \text{sen} 2t + \frac{1}{4} t \text{sen} 2t, \quad C_1, C_2 \in \mathfrak{R}$$

$$\text{f) } x(t) = C_1 \cos 3t + C_2 \text{sen} 3t + e^{3t} \left(\frac{1}{18} t^2 - \frac{1}{27} t + \frac{4}{81} \right), \quad C_1, C_2 \in \mathfrak{R}$$

$$\text{g) } x(t) = e^{3t} \left(C_1 + C_2 t - \frac{2}{9} \cos 3t \right), \quad C_1, C_2 \in \mathfrak{R}$$

$$\text{h) } x(t) = K_1 e^{3t} + K_2 e^{2t} + e^t \left(\frac{1}{2} t^2 + \frac{3}{2} t + \frac{1}{4} \right), \quad K_1, K_2 \in \mathfrak{R}$$

$$\text{i) } x(t) = K_1 \cos t + K_2 \text{sen} t + \cos t \cdot \log|\cos t| + t \text{sen} t, \quad K_1, K_2 \in \mathfrak{R}$$

2.5

$$x(t) = C_1 e^{-2t} + C_2 t e^{-2t} + C_3 \cos t + C_4 \text{sen} t$$

$$\text{a) } x(0) = C_3 \quad x'(0) = C_4 \quad x''(0) = -C_3 \quad x'''(0) = -C_4$$

$$\text{b) } x(0) = C_1 \quad x'(0) = -2C_1 + C_2 \quad x''(0) = 4(C_1 - C_2) \quad x'''(0) = -8C_1 + 12C_2$$

c) impossível

$$\text{d) } x(0) = C_1 + C_3 \quad x'(0) = -2C_1 + C_2 + C_4 \quad x''(0) = 4C_1 - 4C_2 - C_3 \quad x'''(0) = -8C_1 + 12C_2 - C_4$$

$$\text{2.6 } x(t) = C_1 \cos t + C_2 \text{sen} t, \quad C_1, C_2 \in \mathfrak{R}$$

$$\text{2.7 b) } w(t) = At^{-3} + B \quad A \neq 0, \quad B \in \mathfrak{R}$$

$$c) x(t) = -4t^{-1} + t^3/4 \quad \forall_{t>0}$$

3. Sistemas de equações diferenciais

$$3.1 \begin{cases} x_1(t) = C_1 e^t + C_2 t e^t \\ x_2(t) = C_2 e^t \end{cases} \quad C_1, C_2 \in \mathfrak{R}$$

3.2

3.3

$$a) x'' - 6x' + 5x = 0$$

$$b) \begin{cases} x(t) = \frac{1}{4}(e^t(5 - y_0) + e^{5t}(y_0 - 1)) \\ y(t) = \frac{1}{4}(e^t(5 - y_0) + 5e^{5t}(y_0 - 1) - 4t) \end{cases}$$

$$c) y_0 = 1$$

$$3.4 \begin{cases} x_1(t) = C_1 \cos t - C_2 \sin t + 1 \\ x_2(t) = C_1 \sin t - C_2 \cos t + t \end{cases} \quad C_1, C_2 \in \mathfrak{R}$$

$$3.5 a_{ii} < 0 \quad \forall_{i=1, \dots, n}$$

$$3.6 a = b = 1 \quad c = 3$$

$$3.7 \text{ por exemplo } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ ou } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$3.8 A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

3.9

$$a) \begin{cases} x_1(t) = e^t \\ x_2(t) = e^t(\cos t - \sin t) \\ x_3(t) = e^t(\cos t + \sin t) \end{cases}$$

$$\text{b) } \begin{cases} x_1(t) = e^{-t} \cos t \\ x_2(t) = e^{-t} (2 \cos t + \sin t) \end{cases}$$

$$3.10 \quad x^0 = \begin{bmatrix} x_1^0 \\ 0 \\ x_2^0 \end{bmatrix}$$

3.11

$$\begin{cases} x_1(t) = 3e^{3t} - e^{2t}(2+t) \\ x_2(t) = e^{2t} \\ x_3(t) = 3e^{3t} - 2e^{2t} \end{cases}$$

$$3.12 \quad \begin{cases} x_1(t) = \frac{1}{\cos t} - 1 + \sin t \cdot \tan t - t \sin t + \cos t \cdot \log|\cos t| \\ x_2(t) = \sin t - t \cos t - \sin t \cdot \log|\cos t| \\ x_3(t) = -\sin t \cdot \tan t + t \sin t - \cos t \cdot \log|\cos t| \end{cases}$$

$$3.14 \quad e^{At} = I + A(e^t - 1)$$

$$3.16 \text{ c) } \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \cos 2t + 5 \sin 2t \\ 2 \cos 2t - 3 \sin 2t \end{bmatrix}$$

$$3.17 \quad z(t) = \phi_1(t) \phi_1^{-1}(0) \phi_2(t) \phi_2^{-1}(0) z_0$$

3.18

$$\text{a) } \begin{cases} x_1(t) = e^{-2t} \cos 3t \\ x_2(t) = e^{-2t} \sin 3t \\ x_3(t) = e^{-4t} \end{cases}$$

$$\text{b) } \begin{cases} x_1(t) = te^t \\ x_2(t) = e^t - te^t \\ x_3(t) = e^t \end{cases}$$

$$\text{c) } \begin{cases} x_1(t) = e^t - te^t + t^2 e^t \\ x_2(t) = e^t - te^t \\ x_3(t) = e^t \end{cases}$$

$$3.19 \text{ b) } \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} e^t \\ 2e^t - \frac{9}{4}e^{-4t} + \frac{1}{4} \end{bmatrix}$$

$$3.20 \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{3}e^t + \frac{1}{3}e^{4t} \\ \frac{5}{4}e^{2t} - \frac{1}{4}e^{-2t} \end{bmatrix}$$

3.22 estável: a) e), instável: b), assintoticamente estável: c) d)

$$3.24 \alpha \leq -\frac{3}{2}$$

3.25 estável: c) d), instável: a) b)

3.26 $x(t) = 1$ estável e $x(t) = 0$ instável

3.28

a) $(0,1)$ $(0,-1)$ $(1,0)$ instáveis; $(-1,0)$ estável

b) $(0, k\pi)_{k \in \mathbb{Z}}$ instáveis

c) $(k\pi, 0)_{k \in \mathbb{Z}}$ instáveis

4. Equações com diferenças

4.1

$$\text{a) } x_n = -\frac{3}{2}((-1)^n - 1)$$

$$\text{b) } x_n = 6 \cdot 4^n + \frac{2}{3}(4^n - 1)$$

$$\text{c) } x_n = 1 - (-2)^n$$

4.3

$$\text{a) } x_n = C_1 \cos \frac{n\pi}{2} + C_2 \operatorname{sen} \frac{n\pi}{2} + C_3 n \cos \frac{n\pi}{2} + C_4 n \operatorname{sen} \frac{n\pi}{2} \quad C_i \in \mathfrak{R}, i = 1, 2, 3, 4$$

$$\text{b) } x_n = C_1 + C_2 n - \frac{1}{2} n^2 + \frac{1}{6} n^3 \quad C_i \in \mathfrak{R}, i = 1, 2$$

$$\text{c) } x_n = 2(-2)^n + 4(-3)^n - \frac{3}{2} n(-2)^n$$

$$\text{d) } y_t = C_1 2^t + C_2 3^t + 4 + t \quad C_i \in \mathfrak{R}, i = 1, 2$$

$$\text{e) } y_t = C_1 + C_2 t - \frac{t^2}{2} + \frac{t^3}{6} \quad C_i \in \mathfrak{R}, i = 1, 2$$

$$\text{f) } y_t = C + 2^{t+1} + \frac{t}{6} + \frac{t^2}{2} + \frac{t^3}{3} \quad C \in \mathfrak{R}, \text{ se } \Delta y_t = y_t - y_{t-1}$$

$$y_t = C + 2^t + \frac{t}{6} - \frac{t^2}{2} + \frac{t^3}{3} \quad C \in \mathfrak{R}, \text{ se } \Delta y_t = y_{t+1} - y_t$$

$$4.4 \text{ a) } \frac{1}{1-3x} \quad \text{b) } \frac{1}{1-x^2}$$

$$4.5 \quad x_n = 5(-1)^n - 6(-2)^n$$

5. Integração de funções complexas

5.1

$$\text{a) } 2\pi i \quad \text{b) } \frac{1}{(n-1)r^{n-1}} (1 - e^{-2\pi i(n-1)}), \quad n > 1$$

$$5.2 \quad 10 - \frac{8}{3}i$$

$$5.3 \quad -\frac{15}{64}$$

$$5.4 \quad -\frac{44}{3} - \frac{8}{3}i$$

$$5.5 \text{ a) } 0 \quad \text{b) } 4\pi i$$

5.7 a) 0 b) $2\pi i(e^2 + 2)$

5.8 a) $-2\pi i$ b) 0

5.9 a) $\frac{\pi i}{2}$ b) $2\pi i$

5.11 a) $\overline{D_1(0)}$ b) $D_2(i)$ c) C

5.12 a) $\frac{1}{2-z}$ b) $\frac{e(z-1)}{z-1-e}$

5.13 a) $\sum_{n \geq 0} \frac{e}{n!} (z-1)^n \quad z \in C$ b) $\sum_{n \geq 0} (-1)^n (z-1)^n \quad z \in D_1(1)$

5.14 a) $z = \frac{\pi}{2} + k\pi$ pólos simples $\forall k \in Z$

b) $z = 0$ removível e $z = 1$ pólo simples

c) $z = 0$ pólo de ordem 2 e $z = k$ pólo simples $\forall k \in Z$

d) $z = \frac{\pi}{2} + k\pi$ pólos simples $\forall k \in Z$

e) $z = 1$ removível e $z = \frac{-1 \pm \sqrt{3}i}{2}$ pólos simples

f) $z = i$ removível e $z = 1, -i$ pólos simples

g) $z = 0$ essencial

5.15 $z = 1$ essencial, $\sum_{n \geq 0} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!} (z-1)^{-2n-1} \quad z \in A(1, 0, +\infty)$

5.16 $e \sum_{n \geq 0} \frac{1}{n!} (z-1)^{-n} \quad z \in A(1, 0, +\infty)$

$$5.17 \operatorname{Res}\left(f, \frac{\pi}{2} + k\pi\right) = -1$$

$$5.18 \text{ a) b) } \operatorname{Res}(f, 0) = 0$$

$$5.19 \text{ a) b) } 0 \quad \text{c) } -2\pi i$$

$$5.20 \pi i \left(\frac{e^i}{4} - \frac{e^{3i}}{6} \right)$$

$$5.21 z = 0 \text{ essencial; } \operatorname{Res}(f, 0) = 0; \quad 0$$

$$5.22 \text{ a) } z = 0 \text{ essencial, função analítica em } \mathbb{C} \setminus \{0\}$$

$$\text{b) } \frac{\pi}{12}$$